MATH Math 162A Review: Inner Product

1. Let x, y be non-zero vectors. Prove that

$$\left\| \frac{x}{\|x\|^2} - \frac{y}{\|y\|^2} \right\| = \frac{\|x - y\|}{\|x\| \cdot \|y\|}.$$

Solution:

2. Prove the Cauchy-Schwarz inequality of the following form

$$\left| \int_0^1 f(x)g(x)dx \right|^2 \leq \int_0^1 f(x)^2 dx \cdot \int_0^1 g(x)^2 dx.$$

(historic note: should be more fair to call it "Cauchy-Bunyakovsky-Schwarz inequality". Cauchy proved the version for sums, and Bunyakovsky proved the above version many years prior to Schwarz.)

Solution: